8 Statistical Intervals for a Single Sample

8-1 INTRODUCTION

A **tolerance interval** is another important type of interval estimate. For example, the chemical product viscosity data might be assumed to be normally distributed. We might like to calculate limits that bound 95% of the viscosity values. For a normal distribution, we know that 95% of the distribution is in the interval

$$\mu - 1.96\sigma, \mu + 1.96\sigma \qquad (8\text{-}1)$$

However, this is not a useful tolerance interval because the parameters μ and σ are unknown. Point estimates such as $\overline{x}$ and s can be used in Equation 8-1 for μ and σ. However, we need to account for the potential error in each point estimate to form a tolerance interval for the distribution. The result is an interval of the form

$$\overline{x} - ks, \overline{x} + ks \qquad (8\text{-}2)$$

where k is an appropriate constant (that is larger than 1.96 to account for the estimation error). As for a confidence interval, it is not certain that Equation 8-2 bounds 95% of the distribution, but the interval is constructed so that we have high confidence that it does.

Confidence and tolerance intervals bound unknown elements of a distribution. In this chapter you will learn to appreciate the value of these intervals. A **prediction interval** provides bounds on one (or more) future observations from the population. For example, a

Keep the purpose of the three types of interval estimates clear:

- A confidence interval bounds population or distribution parameters (such as the mean viscosity).
- A tolerance interval bounds a selected proportion of a distribution.
- A prediction interval bounds future observations from the population or distribution.

8-2 CONFIDENCE INTERVAL ON THE MEAN OF A NORMAL DISTRIBUTION, VARIANCE KNOWN

8-2.1 Development of the Confidence Interval and Its Basic Properties

Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from a normal distribution with unknown mean μ and known variance σ^2. From the results of Chapter 5 we know that the sample mean $\overline{X}$ is normally distributed with mean μ and variance σ^2/n. We may **standardize** $\overline{X}$ by subtracting the mean and dividing by the standard deviation, which results in the variable

$$Z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \tag{8-3}$$

The random variable Z has a standard normal distribution.

A **confidence interval** estimate for μ is an interval of the form $l \leq \mu \leq u$, where the end-points l and u are computed from the sample data. Because different samples will produce different values of l and u, these end-points are values of random variables L and U, respectively. Suppose that we can determine values of L and U such that the following probability statement is true:

$$P\{L \leq \mu \leq U\} = 1 - \alpha \tag{8-4}$$

where $0 \leq \alpha \leq 1$. There is a probability of $1 - \alpha$ of selecting a sample for which the CI will contain the true value of μ. Once we have selected the sample, so that $X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n$, and computed l and u, the resulting **confidence interval** for μ is

$$l \leq \mu \leq u \tag{8-5}$$

The end-points or bounds l and u are called the **lower-** and **upper-confidence limits,** respectively, and $1 - \alpha$ is called the **confidence coefficient.**

In our problem situation, because $Z = (\overline{X} - \mu)/(\sigma/\sqrt{n})$ has a standard normal distribution, we may write

$$P\left\{-z_{\alpha/2} \leq \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \leq z_{\alpha/2}\right\} = 1 - \alpha$$

$$P\left\{\overline{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \overline{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right\} = 1 - \alpha \tag{8-6}$$

Confidence Interval on the Mean, Variance Known

If $\overline{x}$ is the sample mean of a random sample of size n from a normal population with known variance σ^2, a $100(1-\alpha)\%$ CI on μ is given by

$$\overline{x} - z_{\alpha/2}\sigma/\sqrt{n} \le \mu \le \overline{x} + z_{\alpha/2}\sigma/\sqrt{n} \tag{8-7}$$

where $z_{\alpha/2}$ is the upper $100\alpha/2$ percentage point of the standard normal distribution.

EXAMPLE 8-1
Metallic Material Transition

ASTM Standard E23 defines standard test methods for notched bar impact testing of metallic materials. The Charpy V-notch (CVN) technique measures impact energy and is often used to determine whether or not a material experiences a ductile-to-brittle transition with decreasing temperature. Ten measurements of impact energy (J) on specimens of A238 steel cut at 60°C are as follows: 64.1, 64.7, 64.5, 64.6, 64.5, 64.3, 64.6, 64.8, 64.2, and 64.3. Assume that impact energy is normally distributed with $\sigma = 1J$. We want to find a 95% CI for μ, the mean impact energy. The required quantities are $z_{\alpha/2} = z_{0.025} = 1.96$, $n = 10$, $\sigma = 1$, and $\overline{x} = 64.46$. The resulting 95% CI is found from Equation 8-7 as follows:

$$\overline{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \overline{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$64.46 - 1.96\frac{1}{\sqrt{10}} \le \mu \le 64.46 + 1.96\frac{1}{\sqrt{10}}$$

$$63.84 \le \mu \le 65.08$$

That is, based on the sample data, a range of highly plausible values for mean impact energy for A238 steel at 60°C is $63.84J \le \mu \le 65.08J$.

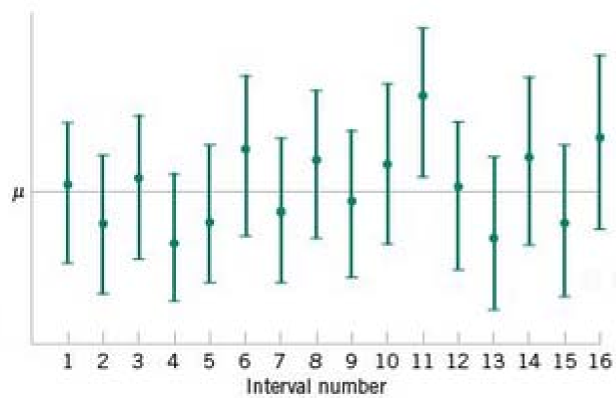

Figure 8-1 Repeated construction of a confidence interval for μ.

Interpreting a Confidence Interval

How does one interpret a confidence interval? In the impact energy estimation problem in Example 8-1 the 95% CI is $63.84 \le \mu \le 65.08$, so it is tempting to conclude that μ is within this interval with probability 0.95. However, with a little reflection, it's easy to see that this cannot be correct; the true value of μ is unknown and the statement $63.84 \le \mu \le 65.08$ is either correct (true with probability 1) or incorrect (false with probability 1). The correct interpretation lies in the realization that a CI is a *random interval* because in the probability statement defining the end-points of the interval (Equation 8-4), L and U are random variables. Consequently, the correct interpretation of a $100(1-\alpha)\%$ CI depends on the relative frequency view of probability. Specifically, if an infinite number of random samples are collected and a $100(1-\alpha)\%$ confidence interval for μ is computed from each sample, $100(1-\alpha)\%$ of these intervals will contain the true value of μ.

Confidence Level and Precision of Estimation

Notice in Example 8-1 that our choice of the 95% level of confidence was essentially arbitrary. What would have happened if we had chosen a higher level of confidence, say, 99%? In fact, doesn't it seem reasonable that we would want the higher level of confidence? At $\alpha = 0.01$, we find $z_{\alpha/2} = z_{0.01/2} = z_{0.005} = 2.58$, while for $\alpha = 0.05$, $z_{0.025} = 1.96$. Thus, the **length** of the 95% confidence interval is

$$2(1.96\sigma/\sqrt{n}) = 3.92\sigma/\sqrt{n}$$

whereas the length of the 99% CI is

$$2(2.58\sigma/\sqrt{n}) = 5.16\sigma/\sqrt{n}$$

Thus, the 99% CI is longer than the 95% CI. This is why we have a higher level of confidence in the 99% confidence interval. Generally, for a fixed sample size n and standard deviation σ, the higher the confidence level, the longer the resulting CI.

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The length of a confidence interval is a measure of the **precision** of estimation. From the preceeding discussion, we see that precision is inversely related to the confidence level. It is desirable to obtain a confidence interval that is short enough for decision-making purposes and that also has adequate confidence. One way to achieve this is by choosing the sample size n to be large enough to give a CI of specified length or precision with prescribed confidence.

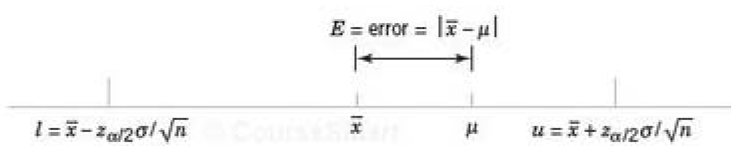

Figure 8-2 Error in estimating μ with $\bar{x}$.

8-2.2 Choice of Sample Size

Sample Size for Specified Error on the Mean, Variance Known

If $\bar{x}$ is used as an estimate of μ, we can be $100(1 - \alpha)\%$ confident that the error $|\bar{x} - \mu|$ will not exceed a specified amount E when the sample size is

$$n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 \tag{8-8}$$

EXAMPLE 8-2
Metallic Material Transition

To illustrate the use of this procedure, consider the CVN test described in Example 8-1, and suppose that we wanted to determine how many specimens must be tested to ensure that the 95% CI on μ for A238 steel cut at 60°C has a length of at most 1.0J. Since the bound on error in estimation E is one-half of the length of the CI, to determine n we use Equation 8-8 with $E = 0.5$, $\sigma = 1$, and $z_{\alpha/2} = 1.96$. The required sample size is 16

$$n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \left[\frac{(1.96)1}{0.5}\right]^2 = 15.37$$

and because n must be an integer, the required sample size is $n = 16$.

One-Sided Confidence Bounds on the Mean, Variance Known

A $100(1 - \alpha)\%$ **upper-confidence bound** for μ is

$$\mu \le u = \bar{x} + z_\alpha \sigma/\sqrt{n} \tag{8-9}$$

and a $100(1 - \alpha)\%$ **lower-confidence bound** for μ is

$$\bar{x} - z_\alpha \sigma/\sqrt{n} = l \le \mu \tag{8-10}$$

EXAMPLE 8-3
One-Sided Confidence Bound

The same data for impact testing from Example 8-1 is used to construct a lower, one-sided 95% confidence interval for the mean impact energy. Recall that $\bar{x} = 64.46$, $\sigma = 1J$, and $n = 10$. The interval is

$$\bar{x} - z_\alpha \frac{\sigma}{\sqrt{n}} \le \mu$$

$$64.46 - 1.64\frac{1}{\sqrt{10}} \le \mu$$

$$63.94 \le \mu$$

The lower limit for the two-sided interval in Example 8-1 was 63.84. Because $z_\alpha < z_{\alpha/2}$, the lower limit of a one-sided interval is always greater than the lower limit of a two-sided interval of equal confidence. The one-sided interval does not bound μ from above so that it still achieves 95% confidence with a slightly greater lower limit. If our interest is only in the lower limit for μ, then the one-sided interval is preferred because it provides equal confidence with a greater lower limit. Similarly, a one-sided upper limit is always less than a two-sided upper limit of equal confidence.

8-3 CONFIDENCE INTERVAL ON THE MEAN OF A NORMAL DISTRIBUTION, VARIANCE UNKNOWN

8-3.1 *t* Distribution

***t* Distribution**

Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal distribution with unknown mean μ and unknown variance σ^2. The random variable

$$T = \frac{\overline{X} - \mu}{S/\sqrt{n}} \tag{8-15}$$

has a t distribution with $n - 1$ degrees of freedom.

The t probability density function is

$$f(x) = \frac{\Gamma[(k+1)/2]}{\sqrt{\pi k}\,\Gamma(k/2)} \cdot \frac{1}{[(x^2/k) + 1]^{(k+1)/2}} \qquad -\infty < x < \infty \tag{8-16}$$

where k is the number of degrees of freedom. The mean and variance of the t distribution are zero and $k/(k-2)$ (for $k > 2$), respectively.

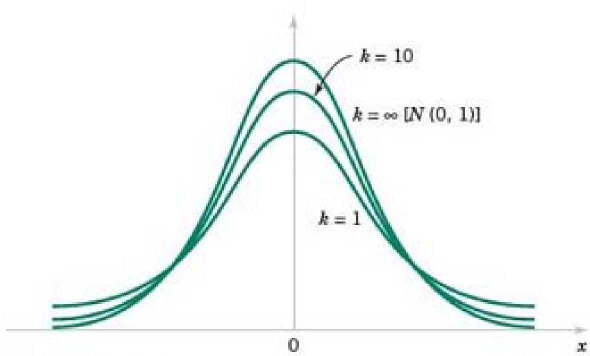

Figure 8-4 Probability density functions of several t distributions.

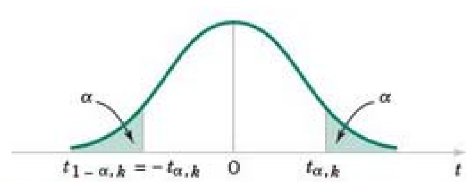

Figure 8-5 Percentage points of the t distribution.

$$P(T_{10} > t_{0.05,10}) = P(T_{10} > 1.812) = 0.05$$

8-3.2 t Confidence Interval on μ

It is easy to find a $100(1-\alpha)$ percent confidence interval on the mean of a normal distribution with unknown variance by proceeding essentially as we did in Section 8-2.1. We know that the distribution of $T = (\overline{X} - \mu)/(S/\sqrt{n})$ is t with $n-1$ degrees of freedom. Letting $t_{\alpha/2,n-1}$ be the upper $100\alpha/2$ percentage point of the t distribution with $n-1$ degrees of freedom, we may write:

$$P(-t_{\alpha/2,n-1} \leq T \leq t_{\alpha/2,n-1}) = 1 - \alpha$$

or

$$P\left(-t_{\alpha/2,n-1} \leq \frac{\overline{X} - \mu}{S/\sqrt{n}} \leq t_{\alpha/2,n-1}\right) = 1 - \alpha$$

Rearranging this last equation yields

$$P(\overline{X} - t_{\alpha/2,n-1}S/\sqrt{n} \leq \mu \leq \overline{X} + t_{\alpha/2,n-1}S/\sqrt{n}) = 1 - \alpha \tag{8-17}$$

This leads to the following definition of the $100(1-\alpha)$ percent two-sided confidence interval on μ.

Confidence Interval on the Mean, Variance Unknown

If $\bar{x}$ and s are the mean and standard deviation of a random sample from a normal distribution with unknown variance σ^2, a **$100(1-\alpha)$ percent confidence interval on μ** is given by

$$\bar{x} - t_{\alpha/2,n-1}s/\sqrt{n} \leq \mu \leq \bar{x} + t_{\alpha/2,n-1}s/\sqrt{n} \tag{8-18}$$

where $t_{\alpha/2,n-1}$ is the upper $100\alpha/2$ percentage point of the t distribution with $n-1$ degrees of freedom.

One-sided confidence bounds on the mean of a normal distribution are also of interest and are easy to find. Simply use only the appropriate lower or upper confidence limit from Equation 8-18 and replace $t_{\alpha/2,n-1}$ by $t_{\alpha,n-1}$.

EXAMPLE 8-5
Alloy Adhesion

An article in the journal *Materials Engineering* (1989, Vol. II, No. 4, pp. 275–281) describes the results of tensile adhesion tests on 22 U-700 alloy specimens. The load at specimen failure is as follows (in megapascals):

19.8	10.1	14.9	7.5	15.4	15.4
15.4	18.5	7.9	12.7	11.9	11.4
11.4	14.1	17.6	16.7	15.8	
19.5	8.8	13.6	11.9	11.4	

The sample mean is $\bar{x} = 13.71$, and the sample standard deviation is $s = 3.55$. Figures 8-6 and 8-7 show a box plot and a normal probability plot of the tensile adhesion test data, respectively. These displays provide good support for the assumption that the population is normally distributed. We want to find a 95% CI on μ. Since $n = 22$, we have $n - 1 = 21$ degrees of freedom for t, so $t_{0.025,21} = 2.080$. The resulting CI is

$$\bar{x} - t_{\alpha/2,n-1}s/\sqrt{n} \le \mu \le \bar{x} + t_{\alpha/2,n-1}s/\sqrt{n}$$

$$13.71 - 2.080(3.55)/\sqrt{22} \le \mu \le 13.71 + 2.080(3.55)/\sqrt{22}$$

$$13.71 - 1.57 \le \mu \le 13.71 + 1.57$$

$$12.14 \le \mu \le 15.28$$

The CI is fairly wide because there is a lot of variability in the tensile adhesion test measurements.

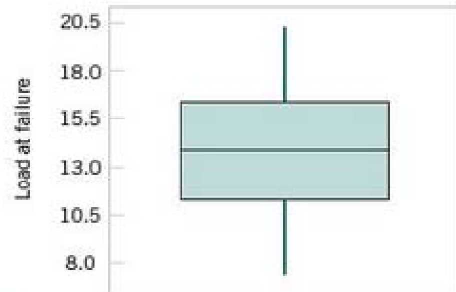

Figure 8-6 Box and whisker plot for the load at failure data in Example 8-5.

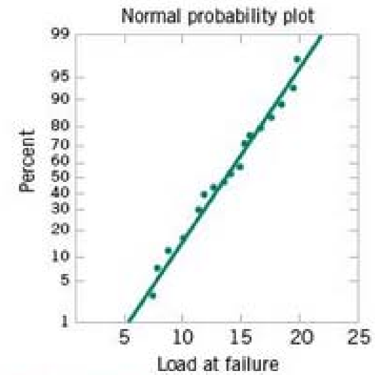

Figure 8-7 Normal probability plot of the load at failure data from Example 8-5.

8-4 CONFIDENCE INTERVAL ON THE VARIANCE AND STANDARD DEVIATION OF A NORMAL DISTRIBUTION

Sometimes confidence intervals on the population variance or standard deviation are needed. When the population is modeled by a normal distribution, the tests and intervals described in this section are applicable. The following result provides the basis of constructing these confidence intervals.

χ^2 Distribution

Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal distribution with mean μ and variance σ^2, and let S^2 be the sample variance. Then the random variable

$$X^2 = \frac{(n-1)\,S^2}{\sigma^2} \tag{8-19}$$

has a chi-square (χ^2) distribution with $n - 1$ degrees of freedom.

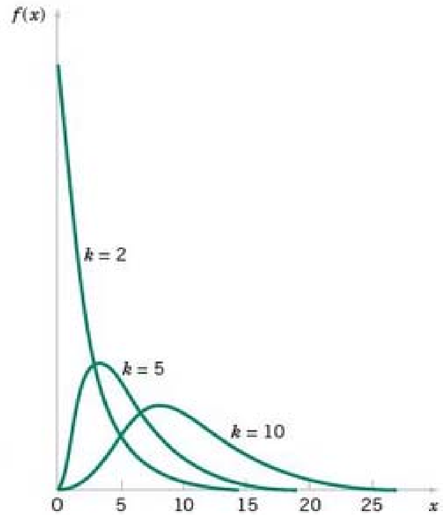

Figure 8-8 Probability density functions of several χ^2 distributions.

The probability density function of a χ^2 random variable is

$$f(x) = \frac{1}{2^{k/2}\Gamma(k/2)} x^{(k/2)-1} e^{-x/2} \qquad x > 0 \tag{8-20}$$

The **percentage points** of the χ^2 distribution are given in Table IV of the Appendix. Define $\chi^2_{\alpha,k}$ as the percentage point or value of the chi-square random variable with k degrees of freedom such that the probability that X^2 exceeds this value is α. That is,

$$P(X^2 > \chi^2_{\alpha,k}) = \int_{\chi^2_{\alpha,k}}^{\infty} f(u)\, du = \alpha$$

This probability is shown as the shaded area in Fig. 8-9(a). To illustrate the use of Table IV, note that the areas α are the column headings and the degrees of freedom k are given in the left column. Therefore, the value with 10 degrees of freedom having an area (probability) of 0.05

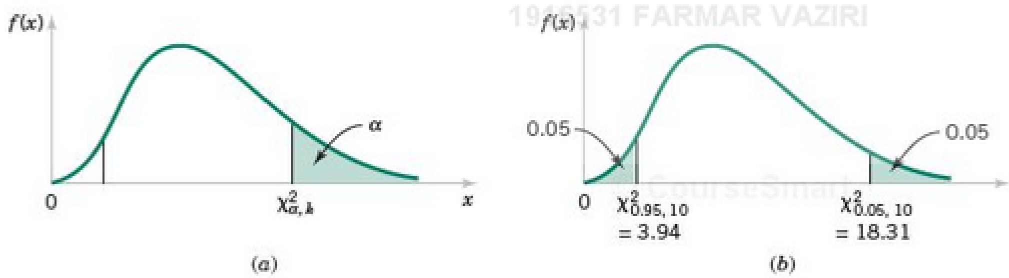

Figure 8-9 Percentage point of the χ^2 distribution. (a) The percentage point $\chi^2_{\alpha,k}$. (b) The upper percentage point $\chi^2_{0.05,10} = 18.31$ and the lower percentage point $\chi^2_{0.95,10} = 3.94$.

to the right is $\chi^2_{0.05,10} = 18.31$. This value is often called an **upper** 5% point of chi-square with 10 degrees of freedom. We may write this as a probability statement as follows:

$$P(X^2 > \chi^2_{0.05,10}) = P(X^2 > 18.31) = 0.05$$

Conversely, a **lower** 5% point of chi-square with 10 degrees of freedom would be $\chi^2_{0.95,10} = 3.94$ (from Appendix A). Both of these percentage points are shown in Figure 8-9(b).

The construction of the $100(1-\alpha)\%$ CI for σ^2 is straightforward. Because

$$X^2 = \frac{(n-1)S^2}{\sigma^2}$$

is chi-square with $n-1$ degrees of freedom, we may write

$$P(\chi^2_{1-\alpha/2,n-1} \le X^2 \le \chi^2_{\alpha/2,n-1}) = 1-\alpha$$

so that

$$P\left(\chi^2_{1-\alpha/2,n-1} \le \frac{(n-1)S^2}{\sigma^2} \le \chi^2_{\alpha/2,n-1}\right) = 1-\alpha$$

This last equation can be rearranged as

$$P\left(\frac{(n-1)S^2}{\chi^2_{\alpha/2,n-1}} \le \sigma^2 \le \frac{(n-1)S^2}{\chi^2_{1-\alpha/2,n-1}}\right) = 1-\alpha$$

This leads to the following definition of the confidence interval for σ^2.

Confidence Interval on the Variance

If s^2 is the sample variance from a random sample of n observations from a normal distribution with unknown variance σ^2, then a **$100(1-\alpha)\%$ confidence interval on σ^2 is**

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}} \quad (8\text{-}21)$$

where $\chi^2_{\alpha/2,n-1}$ and $\chi^2_{1-\alpha/2,n-1}$ are the upper and lower $100\alpha/2$ percentage points of the chi-square distribution with $n-1$ degrees of freedom, respectively. A **confidence interval for σ** has lower and upper limits that are the square roots of the corresponding limits in Equation 8-21.

It is also possible to find a $100(1-\alpha)\%$ lower confidence bound or upper confidence bound on σ^2.

One-Sided Confidence Bounds on the Variance

The $100(1-\alpha)\%$ lower and upper confidence bounds on σ^2 are

$$\frac{(n-1)s^2}{\chi^2_{\alpha,n-1}} \le \sigma^2 \quad \text{and} \quad \sigma^2 \le \frac{(n-1)s^2}{\chi^2_{1-\alpha,n-1}} \quad (8\text{-}22)$$

respectively.

EXAMPLE 8-6
Detergent Filling

An automatic filling machine is used to fill bottles with liquid detergent. A random sample of 20 bottles results in a sample variance of fill volume of $s^2 = 0.0153$ (fluid ounces)^2. If the variance of fill volume is too large, an unacceptable proportion of bottles will be under- or overfilled. We will assume that the fill volume is approximately normally distributed. A 95% upper-confidence interval is found from Equation 8-22 as follows:

$$\sigma^2 \le \frac{(n-1)s^2}{\chi^2_{0.95,19}}$$

or

$$\sigma^2 \le \frac{(19)0.0153}{10.117} = 0.0287 \text{ (fluid ounce)}^2$$

This last expression may be converted into a confidence interval on the standard deviation σ by taking the square root of both sides, resulting in

$$\sigma \le 0.17$$

Therefore, at the 95% level of confidence, the data indicate that the process standard deviation could be as large as 0.17 fluid ounce.

8-7 TOLERANCE AND PREDICTION INTERVALS

8-7.1 Prediction Interval for a Future Observation

In some problem situations, we may be interested in predicting a future observation of a variable. This is a different problem than estimating the mean of that variable, so a confidence interval is not appropriate. In this section we show how to obtain a $100(1-\alpha)\%$ **prediction interval** on a future value of a normal random variable.

Table 8-1 The Roadmap for Constracting Confidence Intervals and Performing Hypothesis Tests, One-Sample Case

Parameter to Be Bounded by the Confidence Interval or Tested with a Hypothesis?	Symbol	Other Parameters?	Confidence Interval Section	Hypothesis Test Section	Comments
Mean of normal distribution	μ	Standard deviation σ known	8-2	9-2	
Mean of arbitrary distribution with large sample size	μ	Sample size large enough that central limit theorem applies and σ is essentially known	8-2.5	9-2.5	Large sample size is often taken to be $n \geq 40$
Mean of normal distribution	μ	Standard deviation σ unknown and estimated	8-3	9-3	
Variance (or standard deviation) of normal distribution	σ^2	Mean μ unknown and estimated	8-4	9-4	
Population Proportion	p	None	8-5	9-5	

Prediction Interval

A $100(1-\alpha)\%$ prediction interval on a single future observation from a normal distribution is given by

$$\bar{x} - t_{\alpha/2,n-1}\, s\sqrt{1+\frac{1}{n}} \leq X_{n+1} \leq \bar{x} + t_{\alpha/2,n-1}\, s\sqrt{1+\frac{1}{n}} \qquad (8\text{-}29)$$

EXAMPLE 8-9
Alloy Adhesion

Reconsider the tensile adhesion tests on specimens of U-700 alloy described in Example 8-5. The load at failure for $n = 22$ specimens was observed, and we found that $\bar{x} = 13.71$ and $s = 3.55$. The 95% confidence interval on μ was $12.14 \leq \mu \leq 15.28$. We plan to test a twenty-third specimen.

A 95% prediction interval on the load at failure for this specimen is

$$\bar{x} - t_{\alpha/2,n-1}s\sqrt{1+\frac{1}{n}} \le X_{n+1} \le \bar{x} + t_{\alpha/2,n-1}s\sqrt{1+\frac{1}{n}}$$

$$13.71 - (2.080)3.55\sqrt{1+\frac{1}{22}} \le X_{23} \le 13.71 + (2.080)3.55\sqrt{1+\frac{1}{22}}$$

$$6.16 \le X_{23} \le 21.26$$

Notice that the prediction interval is considerably longer than the CI.

8-7.2 Tolerance Interval for a Normal Distribution

Consider a population of semiconductor processors. Suppose that the speed of these processors has a normal distribution with mean $\mu = 600$ megahertz and standard deviation $\sigma = 30$ megahertz. Then the interval from $600 - 1.96(30) = 541.2$ to $600 + 1.96(30) = 658.8$ megahertz captures the speed of 95% of the processors in this population because the interval from -1.96 to 1.96 captures 95% of the area under the standard normal curve. The interval from $\mu - z_{\alpha/2}\sigma$ to $\mu + z_{\alpha/2}\sigma$ is called a **tolerance interval.**

If μ and σ are unknown, we can use the data from a random sample of size n to compute $\bar{x}$ and s, and then form the interval $(\bar{x} - 1.96s, \bar{x} + 1.96s)$. However, because of sampling variability in $\bar{x}$ and s, it is likely that this interval will contain less than 95% of the values in the population. The solution to this problem is to replace 1.96 by some value that will make the proportion of the distribution contained in the interval 95% with some level of confidence. Fortunately, it is easy to do this.

Tolerance Interval

A **tolerance interval** for capturing at least γ% of the values in a normal distribution with confidence level $100(1-\alpha)$% is

$$\bar{x} - ks, \qquad \bar{x} + ks$$

where k is a tolerance interval factor found in Appendix Table XII. Values are given for $\gamma = 90\%$, 95%, and 99% and for 90%, 95%, and 99% confidence.

EXAMPLE 8-10
Alloy Adhesion

Let's reconsider the tensile adhesion tests originally described in Example 8-5. The load at failure for $n = 22$ specimens was observed, and we found that $\bar{x} = 13.71$ and $s = 3.55$. We want to find a tolerance interval for the load at failure that includes 90% of the values in the population with 95% confidence. From Appendix Table XII the tolerance factor k for $n = 22$, $\gamma = 0.90$, and 95% confidence is $k = 2.264$. The desired tolerance interval is

$$(\bar{x} - ks, \bar{x} + ks) \quad \text{or} \quad [13.71 - (2.264)3.55, 13.71 + (2.264)3.55]$$

which reduces to (5.67, 21.74). We can be 95% confident that at least 90% of the values of load at failure for this particular alloy lie between 5.67 and 21.74 megapascals.