Solutions
Chapter 5

Problem 1

Series limit for the Lyman series is

$$\nu_\infty = cR_H \left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) = cR_H = 3.3 \times 10^{15} \text{ Hz.}$$

Series limit for the Balmer series is

$$\nu_\infty = cR_H \left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) = cR_H/4 = 8.2 \times 10^{14} \text{ Hz.}$$

Problem 2

Lowest frequency for the Lyman seris is

$$\nu_l = cR_H \left(\frac{1}{1^2} - \frac{1}{2^2}\right) = cR_H(3/4) = 2.4 \times 10^{15} \text{ Hz.}$$

Lowest frequency for the Balmer series is

$$\nu_l = cR_H \left(\frac{1}{2^2} - \frac{1}{3^2}\right) = cR_H(5/36) = 4.6 \times 10^{14} \text{ Hz.}$$

Problem 3

The difference between the hydrogen atom and the singly ionized helium atom is in nucleus – there are two protons instead of just one. Hence, the product of charges in the potential energy function is $2e^2$ instead of e^2. So, in the final formula for the hydrogen Rydberg constant R_H, e^2 must be replaced by $2e^2$ to obtain the helium Rydberg constant R_{He}.

$$R_{He} = \frac{m(2e^2)^2}{4\pi c(4\pi\epsilon_0)^2\hbar^3} = 4R_H = 4.387 \times 10^7 \text{m}^{-1}.$$

Problem 4

The equivalent of the Balmer series for the singly ionized helium atom has the following frequencies.

$$\nu = cR_{He} \left(\frac{1}{2^2} - \frac{1}{n^2}\right), \quad n = 1, 2, 3, \ldots$$

So its series limit is

$$\nu_\infty = cR_{He}\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) = cR_{He}/4 = cR_H = 3.3 \times 10^{15} \text{ Hz.}$$

The lowest frequency is

$$\nu_l = cR_{He}\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = cR_{He}(5/36) = cR_H(5/9) = 1.8 \times 10^{15} \text{ Hz.}$$

Problem 5

The difference between such an atom and the hydrogen atom is the number of protons in the nucleus – Ze instead of e. So the product of charges in the potential energy formula is Ze^2. Hence, in the formula for hydrogen Rydberg constant (R_H), e^2 must be substituted by Ze^2 to obtain the constant for atoms with atomic number Z. This gives

$$R_Z = \frac{m(Ze^2)^2}{4\pi c(4\pi\epsilon_0)^2\hbar^3} = Z^2 R_H = Z^2(1.09678 \times 10^7)\text{m}^{-1}.$$

Problem 6

l	m_l
4	$-4, -3, -2, -1, 0, 1, 2, 3, 4.$
3	$-3, -2, -1, 0, 1, 2, 3.$
2	$-2, -1, 0, 1, 2.$
1	$-1, 0, 1.$
0	$0.$

Problem 7

The degree of degeneracy for each value of l is $2l + 1$. Hence, the degree of degeneracy for a specific value of n is (using the formula for the sum of an arithmetic series)

$$d = \sum_{l=0}^{n-1}(2l+1) = 2\sum_{l=0}^{n-1} l + \sum_{l=0}^{n-1} 1 = 2\frac{(n-1)n}{2} + n = n^2.$$

Problem 8

$$(n = 1, l = 0, m_l = 0, m_s = +1/2), \quad (n = 1, l = 0, m_l = 0, m_s = -1/2),$$

$$
\begin{array}{ll}
(n=2, l=0, m_l=0, m_s=+1/2), & (n=2, l=0, m_l=0, m_s=-1/2), \\
(n=2, l=1, m_l=+1, m_s=+1/2), & (n=2, l=1, m_l=+1, m_s=-1/2), \\
(n=2, l=1, m_l=0, m_s=+1/2), & (n=2, l=1, m_l=0, m_s=-1/2), \\
(n=2, l=1, m_l=-1, m_s=+1/2), & (n=2, l=1, m_l=-1, m_s=-1/2), \\
(n=3, l=0, m_l=0, m_s=\pm 1/2). &
\end{array}
$$