STATE UNIVERSITY OF NEW YORK
New Paltz, New York.

General Physics 2
First Exam

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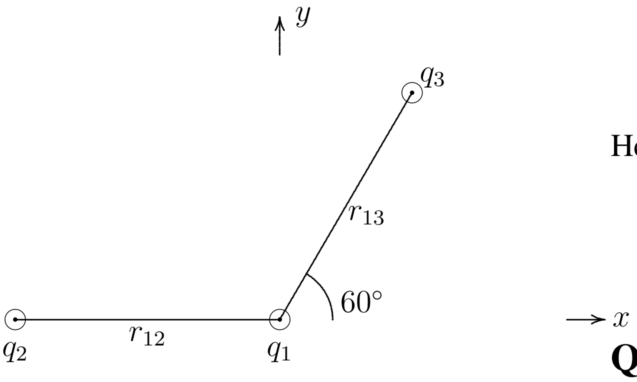
Solutions

Constants and Formulas

$$
\begin{aligned}
k = \frac{1}{4\pi\epsilon_0} &= 9.0 \times 10^9 \quad Nm^2/C^2. \\
F &= k\frac{|q_1||q_2|}{r^2} \\
E &= k\frac{|q|}{r^2} \\
\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} &= q_e/\epsilon_0 \\
V &= k\frac{q}{r}
\end{aligned}
$$

Name______________________________

Problem I



The figure above shows three point charges: $q_1 = 4.0\mu$C, $q_2 = 3.0\mu$C, and $q_3 = 5.0\mu$C. The distances are: $r_{12} = 5.0$m and $r_{13} = 5.0$m. The x and y coordinates are as shown and q_1 is at the origin.

Solution

Question 1

The magnitude of the force is

$$F_{12} = \frac{kq_1q_2}{r_{12}^2} = 4.3 \times 10^{-3}\ \text{N}.$$

The direction is along the positive x direction. Hence,

$$\vec{\mathbf{F}}_{12} = 4.3 \times 10^{-3}\hat{\mathbf{i}}\ \text{N}.$$

Question 2

The magnitude of the force is

$$F_{13} = \frac{kq_1q_3}{r_{13}^2} = 7.2 \times 10^{-3}\ \text{N}.$$

The direction is $-120°$ from the positive x direction. Hence,

$$F_{13x} = F_{13}\cos(-120°) = -3.6 \times 10^{-3}\ \text{N}.$$

and

$$F_{13y} = F_{13}\sin(-120°) = -6.2 \times 10^{-3}\ \text{N}.$$

Hence,

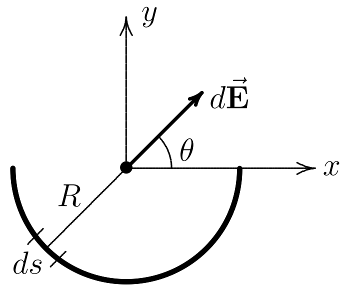
$$\vec{\mathbf{F}}_{13} = -3.6 \times 10^{-3}\hat{\mathbf{i}} - 6.2 \times 10^{-3}\hat{\mathbf{j}}\ \text{N}$$

Question 3

$$\vec{\mathbf{F}}_1 = \vec{\mathbf{F}}_{12} + \vec{\mathbf{F}}_{13}$$

$$= 0.7 \times 10^{-3}\hat{\mathbf{i}} - 6.2 \times 10^{-3}\hat{\mathbf{j}}\ \text{N}.$$

Problem II



The figure above shows a thin wire bent in the shape of a half circle of radius R with center at the origin of the coordinate system shown. It has a uniform charge density of λ C/m. Other symbols to be used are shown in the figure.

Solution

Question 4

The magnitude of $d\vec{\mathbf{E}}$ produced by a small element of charge $dq = \lambda ds$ is

$$dE = \frac{kdq}{R^2} = \frac{k\lambda ds}{R^2}$$

The x component of this is

$$dE_x = dE\cos\theta = \frac{k\lambda\cos\theta\ ds}{R^2}$$

Question 5

Similarly, the y component is

$$dE_y = dE\sin\theta = \frac{k\lambda\sin\theta\ ds}{R^2}$$

Question 6

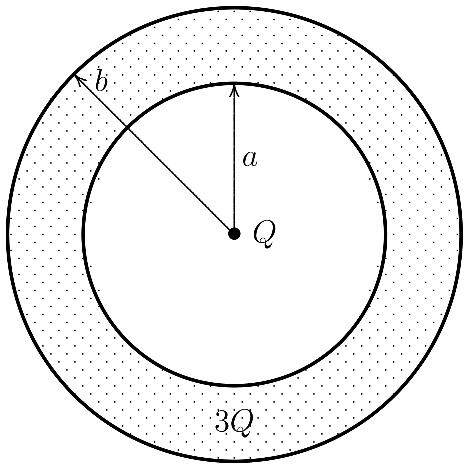
As $ds = R\ d\theta$,

$$dE_y = \frac{k\lambda\sin\theta\ d\theta}{R}$$

Integrating dE_y, we get

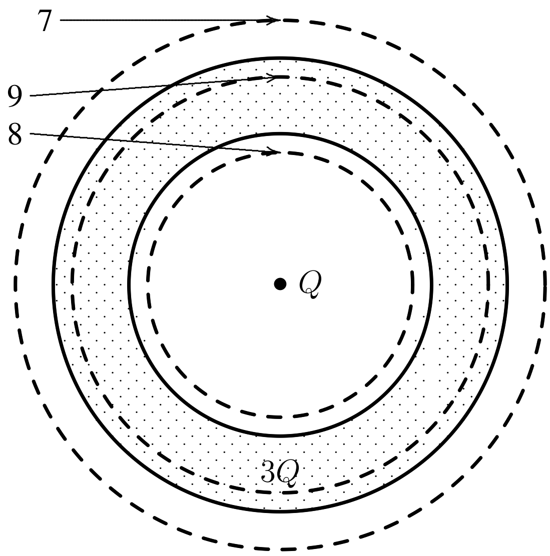
$$E_y = \int dE_y = \frac{k\lambda}{R}\int_0^{\pi}\sin\theta d\theta = \frac{2k\lambda}{R}$$

Problem III



The figure above shows a conducting spherical shell (inner radius a and outer radius b). It has a charge of $3Q$ placed on it. A point charge of Q is placed at the center of the sphere. Q is positive. The system is under static conditions.

Solution



For a spherical Gaussian surface of radius r

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint E dA = E \oint dA = E(4\pi r^2)$$

Question 7

For this case, the Gaussian surface is shown in the figure. The enclosed charge for this surface is $3Q + Q = 4Q$. Hence,

$$E(4\pi r^2) = 4Q/\epsilon_0$$

and $E = 4Q/(4\pi\epsilon_0 r^2)$.

Question 8

For this case, the Gaussian surface is shown in the figure. The enclosed charge for this surface is Q. Hence,

$$E(4\pi r^2) = Q/\epsilon_0$$

and $E = Q/(4\pi\epsilon_0 r^2)$.

Question 9

For this case, the Gaussian surface is shown in the figure. The enclosed charge is $Q_i + Q$, where Q_i is the charge on the inner surface of the shell. But $E = 0$ for all points on this surface as it is completely inside a conductor. Hence,

$$0 = (Q_i + Q)/\epsilon_0$$

and

$$Q_i = -Q$$

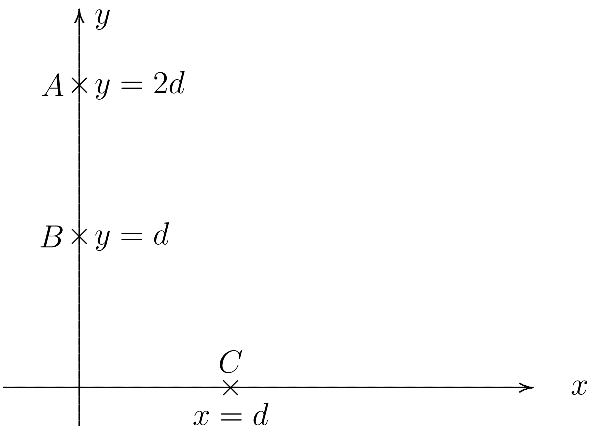
. If Q_o is the charge on the outer surface, then the total charge on the shell is

$$Q_i + Q_o = 3Q$$

Hence,

$$Q_o = 3Q - Q_i = 4Q$$

Problem IV



The figure above shows a coordinate system with three positions (A, B and C) marked with a '$\times$'. A and B are on the y axis at $y = 2d$ and $y = d$ respectively. C is on the x axis at $x = d$.

Solution

Question 10

$$V = \frac{kQ}{2d} + \frac{k(-Q)}{d} = \frac{-kQ}{2d}$$

Question 11

$$V = \frac{kQ}{d} + \frac{k(-Q)}{d} = 0.$$

Question 12

As the two charges each produce non-zero electric fields, one in the x direction and the other in the y direction, they cannot cancel each other. Hence,

$$\vec{\mathbf{E}} \neq 0.$$